SIMULATION OF SPATIAL FORM OF URBAN SYSTEMS BY DIFFUSION METHODS (PART 1)

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Abstract: Modeling method for forecasting of social-economic processes in accordance with methodology of applique fractal crystals growth methods in fuzzy attraction potential field was proposed. Impact of model empirical parameters on appearance of fractal structure fluctuations in the form of creating additional aggregation centers was investigated. Computer experiments give a possibility to simulate structures which are well correlated with experimental data received.

Key words: potential of attractiveness, fractal, fuzzy logic, molecular dynamics

Introduction

Stormy development of all tourism activities at the end of XX century influence on the concentrating of big capital in tourist industry – hotel chains appear, building of tourist centers begins, transport infrastructure and restaurant services grow up.

Investing considerable funds in tourism development, a large capital requires a maximal income as soon as possible. Natural landscape and local population are perceived only as labors for goal achievement. No wonder that mass growth of non regulation visits of the prominent natural complexes has the negative influence on them and also on a local social and cultural environment: the rare plants are destroyed, trees are cut down, reservoirs are contaminated, populations of many types of animals disappeared or considerably decreased. Such kind of tourism which got the name of “cruel tourism” took place in the last decades in many countries.

Information stated above testifies that the tourism development planning and prognostication are very important nowadays. To solve this problem various mathematical methods and models should be used. The usage of mathematical
methods in economic researches gives us the possibility to solve specific tasks with the construction of forecasting scenarios and a possibility to foresee forming and development of the complex socio-economic processes.

Development of GPS technologies and various international programs of the space sounding and Earth photographing, digital cards creations allowed everyone to be freely oriented on locality, get topographical data and information about the location, plan the route, to get the locality images from space and other (Cryuchkov, 2006). These data open up large possibilities for scientists to conduct researches in the sphere of GIS technologies, architecture, sociology, economies and other science industries. The imprints of most settlements which are done from space remind the aggregated crystal growth on the definite centers (entertaining, recreation, industrial and others centers) and deformed by the definite potential field. In this paper the methodology of crystals fractal growth appliqué methods in the fuzzy potential attraction field for forecasting of poorly controlled social processes on the example of settlements geometrical form forecasting is proposed. Advantages and lacks of the modified theory of diffuse-limited aggregation (DLA) and «Accidental rain» (AR) are discovered and the combination algorithm of this theories to remove basic failings and to perform maximal use of advantages of the lasts are offered.

The purpose of this research is the creation and the approbation of the methodology of crystals fractal growth appliqué methods in the fuzzy potential attraction field for social processes forecasting on the example of settlements geometrical form forecasting.

Research actuality consists in conception development of the poorly controlled social processes prognostication such as the cities and settlements growth, related to active development of green tourism, creation of concomitant infrastructure, people division on segments after general interests, work, rest, and others based on crystals fractal growth methods well known in solid physics in combination with fuzzy logic theory.

Forming of structures with growing surfaces are present in the wide range of the phenomena which are actively studied by science, in particular crystals growth under various conditions, snowflake evolution in an atmosphere, the directed solidification in some processes that act important part in metallurgy (Pietronero, 1988).

Settlements growth is characterized by the set of features which are present in the physical processes of crystals growth, in particular:
Physical crystal growth begins on a definite center. Such centers could be production enterprises, historical and cultural legacy, tourist-recreation systems (TRS), mountain-skier centers, entertaining centers, beaches and others;

Clusters deformation in physical crystals is conditioned by diffusion in the potential field. The role of the potential field in social processes play the attractiveness field, that depends on distance, infrastructure, innovative-investment climate, relief, legal and other aspects. The field can be built by the fuzzy logic theory.

In the movements process the free particle which creates the accidental moving joins either to the center of cluster or to the before aggregated particles. In obedience to marketing researches (Douroovich, 2003) new recreation objects or new buildings appear in a direct closeness from neighbors, forming quarters, analogue of clusters.

From stated above becomes clear that basic processes of settlements alteration similar to the processes that are observed at crystals growth. It enables to use the approved theories DLA and the «Accidental rain» (Batty, 1996) for the design of socio-economic processes.

Forecasting of settlements growth geometry will allow creating proper infrastructure and communications with a maximal economic value and possibility to foresee the structure of a new building near accrued tourist-recreation systems. In its turn this will allow optimizing strategy of building new TRS’s, defining specialization of separate settlement segments and foresee the money streams of the system (Leonenko, 2005).

However the distinguishing feature of settlements growth is that crystallization (aggregation) takes place not in one center as is observed in the physical phenomena. In the real life there are several crystallization centers and regions within the limits of the explored object and they have almost all the time difficult geometry. The potential attraction field in turn also has a difficult form. In large towns and megalopolises strategy of alteration is formed in obedience to expert estimations and permissions of the proper establishments. Alteration of small settlements carries a probabilistic character and in a prominent measure relies on the attractiveness of definite territory. From foregoing becomes clear that the classic methods of crystal growth like imitation, dendrite and fractal growth should be substantially modified (Pietronero, 1988).
Model of the potential field

Unlike the physical fields it is difficult to formalize the potential attractiveness field of some territory for building using classic mathematic tools. Such behavior is the result of basing of potential territory attractiveness field on human logic and human senses. In addition, the size of the potential attractiveness field relies on the geographical location, locality relief, presence of the proper flora and fauna, temperature conditions, possibilities to form proper transport infrastructure etc. Taking into account the transferred entry parameters the size of the potential field can be described by mathematical fuzzy logic tools. In general the potential $U$ could be presented as:

$$
U = F(a_1, a_2, ..., a_n),
$$

where $a_i$ – entry parameters $F$ – function which is determined be the type of potential.

The type of function and choice of algorithm of the fuzzy conclusion (Mamdani, Sougeno, Tsoucamoto and others (Leonenko, 2005)) relies on the mechanism of construction of fuzzy production rules, that are used in consulting and handling models and in its basis had the knowledge base formed by the specialists-experts of subject domain or got as a result of neural network teaching, educational great number of which, in turn is based on experimental data, as an aggregate of fuzzy predicate rules. Fuzzy logic tools show on advantage in the researches of economic and social processes, in particular at computations of the efficiency integrated indexes (Petrenko, 2006), decision of multicriterion tasks and economy growing competition determination between regions in China (Shengquan, 2006). In previous work (Vyklyuk, 2008) we showed the advantages of using algorithms Mamdani and Sougeno for determination of recreation potential. It was shown, that the results which were received after using these methods correlate well with experts estimations. Therefore in subsequent computations we have used Sougeno with the Gaussian membership functions (Pietronero, 1988). This algorithm was chosen because the presence of experimental knowledge basis gives the possibility of using hybrid neuron networks ANFIS (Adaptive Neuro-Fuzzy Inference System).

Algorithm Sougeno

Let knowledge base contain only two fuzzy rules like:

Rule1: if $x$ is $a_i$ and $y$ is $b_i$, then $z_1 = a_i x + b_i y$,
Rule 2: if \( x \) is \( a_2 \) and \( y \) is \( b_2 \), then \( z_2 = a_2 x + b_2 y \),

1. Fuzziness: the true degrees of every pre-condition for each rule are found.
\[
\mu_{a_1}(x_0), \, \mu_{a_2}(x_0), \, \mu_{b_1}(y_0), \, \mu_{b_2}(y_0).
\]

2. Fuzzy conclusion: the even "cutoffs" for pre-conditions of each rule are found (with the use of min operation)
\[
\alpha_1 = \mu_{a_1}(x_0) \, \mu_{b_2}(y_0),
\]
\[
\alpha_2 = \mu_{a_1}(x_0) \, \mu_{b_2}(y_0).
\]

Then “cutoff” membership function are found
\[
\begin{align*}
z^*_1 &= a_1 x_0 + b_2 y_0, \\
z^*_2 &= a_2 x_0 + b_2 y_0,
\end{align*}
\]

3. The sharp variable value of conclusion is found:
\[
z_0 = \frac{\alpha_1 z^*_1 + \alpha_2 z^*_2}{\alpha_1 + \alpha_2}.
\]

For the calculation of potential attractiveness field form the method of recreational potentials maps construction could be used (Kyfyak, 2007). For this purpose the map of the territory \( T \) is covered by a rectangle \( R = [a,b] \times [c,d] \). Obviously, that a rectangle \( R \) contains the set (territory) \( T \) \((T \subset \Pi)\). Rectangle \( R \) is broken up by a net \( \Delta = \Delta_x \times \Delta_y \), where:

\[
\Delta_x = \bigcup_{k=0}^{N} \{ x_k \},
\]

\[
\Delta_y = \bigcup_{l=0}^{M} \{ y_l \},
\]

\[
x_k = x_o + kh_x, \quad k = 0,N,
\]

\[
y_l = y_0 + lh_y, \quad l = 0,M,
\]
For every knot of the net the entry parameter’s values are determined. Received matrices serve by the entry parameters for fuzzy function of the attractiveness potential field (1). A matrix is the result of calculation which determines the form of territory potential T.

Entry parameters are divided into two types. Their co-ordinates are exactly definite by GPS technologies and those which should be determined with one’s own hand.

The entrance sizes of the first type can be divided into sub-groups. There are locally concentrated and definite by the vectors.

Locally concentrated objects (medical water, history-cultural centers, mountain-skier, parks etc.) act as part of crystallization centers. For determination of the objects attractiveness potential distance to them is used and not the co-ordinates of these objects.

Vectors help to determine transport networks. It is implicit circumstance that habitation alteration, especially in green tourism, move to the already existing motorways. It is confirmed by the numerous settlements pictures from space (Cryuchkov, 2006). While the distance to the roads increases the territory attractiveness for building falls. We suggest using distance to the nearest road and distance by the road to the nearest crystallization center as entry parameters for fuzzy attractiveness potential.

![Figure 1](image.png)

**Figure 1** The triangle co-ordinates for determine the attitude to side \(a\) (\(h'\)) and the attitude basis \((x_{1_{h'}}, y_{1_{h'}})\)
Let the transport networks of the explored territory be set by an array:

\[ w_f(x_{j_1}, y_{j_1}, x_{j_2}, y_{j_2}), \quad f = 1, n, \]  

(14)

where \( n \) – quantity of roads vectors on the explored territory \( x_{j_1}, y_{j_1}, x_{j_2}, y_{j_2} \) – co-ordinates of road vector.

Then distance \( h \) of point with co-ordinates \( x, y \) to the nearest road is determined in obedience to the following reasoning: we will consider a triangle with tops \( A(x, y), B(x_{j_1}, y_{j_1}), C(x_{j_2}, y_{j_2}) \) (Fig. 1.). The sides length is determined as:

\[ a = \sqrt{(x_{j_1} - x_{j_2})^2 - (y_{j_1} - y_{j_2})^2}, \]  

(15)

\[ b = \sqrt{(x - x_{j_1})^2 - (y - y_{j_1})^2}, \]  

(16)

\[ c = \sqrt{(x - x_{j_2})^2 - (y - y_{j_2})^2}. \]  

(17)

The triangle attitude to side \( a \) (segment of road):

\[ h'_f = \frac{2\sqrt{p(p - a)(p - b)(p - c)}}{a}, \]  

(18)

\[ p = \frac{a + b + c}{2}. \]  

(19)

Segments \( a_1 \) and \( a_2 \), that determine distance between the attitude basis to the tops B and C accordingly and co-ordinates of attitude basis \( (x'_{b}, y'_{b}) \) are determined as follows:

\[ a_1 = \sqrt{b^2 - h'^2_f}, \]  

(20)

\[ a_2 = \sqrt{c^2 - h'^2_f}, \]  

(21)

\[ x'_{bh} = x_{j_1} - (x_{j_1} - x_{j_2})\frac{a_1}{a}, \]  

(22)
An attitude (12) is the shortest way to the road in the case if point \((x'_{fh}, y'_{fh})\) lies on the segment of road \(f\). In other case the shortest distance to the road will be determined as:

\[
h_f = \begin{cases} 
  a_1 + a_2 = a, & h'_f, \\
  a_1 + a_2 > a \text{ and } b < c, & b, \\
  a_1 + a_2 > a \text{ and } b > c, & c.
\end{cases}
\]  

Intersection co-ordinates accordingly:

\[
(x_{fh}, y_{fh}) = \begin{cases} 
  a_1 + a_2 = a, & (x'_{fh}, y'_{fh}) \\
  a_1 + a_2 > a \text{ and } b < c, & (x_1, y_1) \\
  a_1 + a_2 > a \text{ and } b > c, & (x_2, y_2)
\end{cases}
\]

Then distance to the nearest road is determined as:

\[
h = \min_{f=1,n}(h_f).
\]

**Model of the modified diffuse-limited aggregation theory (DLA)**

Nowadays there are a lot of models which describe the irreversible particles unions in clusters. Aggregation process is described by the nonlinear differential equation in partial derivative. Solution of these equations come from on analytical and a number of methods and has large complication. One of possible methods studying such questions consists in the study of the model systems which are able to generate such structures. The most known method is DLA model (Pietronero, 1988, Ball, 1984, Witten Jr 1981).

A classic DLA model is very simple: particles with accidental moving in the aggregation process form a cluster. So the particle starts it motion from random point joins either to the point clustering center or to the before aggregated particles. Intensive computer research es showed that the difficult ramified fractals (Pietronero, 1988, Batty, 1996) which have a spherical form appear as a result of such process.

In our case a particle must move in the potential field which has influence on fractal form. To design this motion we could use molecular dynamics methods.
Let us suppose that in the moment of time $t$ a particle is found in a point $(x(t), y(t))$ but moves with speed $(v_x(t), v_y(t))$. So in a projection on an axis the force which operates on a particle is:

$$F_x(t) = -\frac{\partial U}{\partial x(t)} ,$$

$$F_y(t) = -\frac{\partial U}{\partial y(t)} .$$

Considering that during small period of time $\Delta t$ force remains unchanging speeds of particle are calculated in the moment of time $t + \Delta t$:

$$a_x(t) = \frac{F_x(t)}{m} ,$$

$$a_y(t) = \frac{F_y(t)}{m} ,$$

$$v_x(t + \Delta t) = a_x(t)\Delta t + v_x(t) ,$$

$$v_y(t + \Delta t) = a_y(t)\Delta t + v_y(t) ,$$

$$x(t + \Delta t) = \frac{a_x(t)\Delta t^2}{2} + v_x(t)\Delta t + x(t) ,$$

$$y(t + \Delta t) = \frac{a_y(t)\Delta t^2}{2} + v_y(t)\Delta t + y(t) .$$

where $m$ – particle mass (Gould, 1990, Kaplan, 1982).

Lack of such approach is that negative part is acted by the energy conservation law. During close to the crystallization center kinetic energy grows in particles and its speed accordingly. Due to discrete time in many cases a particle flies everywhere and is not aggregated on the crystallization center. The solution of this problem is to reduce discreteness time which influence on increasing computation time. Another solution is normalization particle speed after each
iteration step. However, as computer experiments showed, in many case a particle goes out on a stationary orbit round the aggregation centers.

In a molecular dynamics temperature is considered as a measure of kinetic energy and come forward as a lever of particles middle speed change. So it is possible to decrease particles’ speed if temperature will be decreased too (Kaplan, 1982). In our research it is necessary to develop and prove mechanism of temperature gradual reduction and particle speed growth. It’s clear, that it is outside influencing which has not analogues in nature.

For the correct influencing of the potential field and prevention of sharp speed growth we suggest considering particles motion in an environment that owns viscid friction. As an example there could be the motion of bodies in air, resistance force of which at subsonic speeds is proportional to speed:

\[ F_i = -\beta v, \]  

(35)

where \( \beta \) – resistance coefficient.

Then:

\[ a_x(t) = \frac{F_x(t) - \beta v_x(t)}{m}, \]  

(36)

\[ a_y(t) = \frac{F_y(t) - \beta v_y(t)}{m}. \]  

(37)

As experiments showed, consideration of viscid friction allows evading the afore-mentioned failing.

Figure 2. Growth cluster in DLA model in the case with 2 crystallization centers
Particle aggregation takes place during motion if it runs into the center of cluster or the before aggregated particles. In the case if entry parameters of fuzzy potential which carry maintenance of local limitations aggregation (coast, bog and reservoir) hinder a particle is withdrawn from computation.

Our researches showed that the offered method describes very well the front geometry of settlement growth. However the given method has the substantial lack: crystals growth process in most cases takes place either on one center or on a plate or wire. The same situation is observed at the design of settlement located along a definite curve (sea coast, road). In this case there is a good concordance with present experimental data. However most settlements have the ramified infrastructure network and set of the territorial distributed attractiveness centers, round which settlement is growing. As our computations showed presence even two distributed accretion centers lead to appearance of empty regions in which particles can not get from outside regardless of its trajectory form (Fig.2.). It leads to situation when city center has empty regions without buildings, that is not observed in real world. The indicated lack could be easily removed using the model of the «Accidental rain».

Conclusions

Method for the construction fuzzy potential field of attractiveness is presented. The algorithm of entry parameters calculation for fuzzy model is offered.

The algorithm of fractal growth calculation is presented in the fuzzy potential field by DLA methods. It is showed that accounting elements of molecular dynamics instruments, force of viscid friction and limitations in the DLA model allows adequately describe motion of particle in the fuzzy potential field.

References


